Aerospace Structures Information and Analysis Center

Effect of Electrostatic Field on Film Rupture

Report No. TR-98-03

September 1998

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Operated for the Flight Dynamics Directorate by CSA Engineering, Inc.

FOREWORD

This report was prepared by the Aerospace Structures Information and Analysis Center (ASIAC), which is operated by CSA Engineering, Inc. under contract number F33615-94-C-3200 for the Flight Dynamics Directorate, Wright-Patterson Air Force Base, Ohio. The report presents the work performed under ASIAC Task No. T- 36. The work was sponsored by the Thermal Structures Branch, Structures Division, Air Vehicles Directorate of the Air Force Research Laboratory at WPAFB, Ohio. The technical monitors for the task was Dr. Larry Byrd of the Thermal Structures Branch. The study was performed by Dr. Rama Gorla, Professor of Mechanical Engineering at Cleveland State University, under contract to CSA Engineering Inc.

This technical report covers work accomplished from May 1997 through July 1998.

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EFFECT OF ELECTROSTATIC FIELD ON FILM RUPTURE

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ABSTRACT

The nonlinear thin film rupture has been analyzed by investigating the stability of thin films under the influence of electrostatic field to finite amplitude disturbances. The dynamics of the liquid film is formulated using the Navier Stokes equations including a body force term due to van der Waals attractions. The effect of the electric field is included in the analysis. The governing equation was solved by finite difference method as part of an initial value problem for spatial periodic boundary conditions. A discussion is provided for the effect of the electric field strength on the film rupture.

NOMENCLATURE

A'	Hamaker constant
Ca	Capillary number
d	characteristic film thickness
E^v	electric field in vapor
F	electric field strength
Fr	Froude number
g	gravitational acceleration
H	distance from charged foil to the plate
h	film thickness
K	dimensionless electric field strength
L	length of plate
p	pressure
Re	Reynolds number
t	time
U_0	reference velocity
u,v	velocity components in x and y directions
x,y	coordinate directions parallel and perpendicular to the plate
β	angle made by the plate with horizontal direction
ϕ	electric potential
ξ	aspect ratio

ρ	density of fluid
μ	dynamic viscosity
ν	Kinematic viscosity
ψ	potential function for van der Waals forces
ε	dielectric constant
σ	surface tension

Subscripts

0,1 first order and second order perturbations

INTRODUCTION

The hydrodynamic stability of thin liquid films has attracted much attention in view of many applications in chemical, mechanical and biomedical engineering fields. When a liquid layer becomes ultra thin $(100-1000\text{Å})$, it becomes unstable. The instability is due to the van der Waals potential and results in the rupture of the layer. The rupture of thin liquid films occurs in many industrial applications involving disperse and colloid systems and in biological phenomena. The spontaneous rupture of a liquid film on a planar solid wall was studied by Ruckenstein and Jain[1]. The liquid film was modeled by them as a Navier-Stokes continuum with a body force with a potential due to the van der Waals interactions. They used lubrication approximation to obtain the linear dynamic instability results. From this analysis, one can obtain rough estimates for the rupture time, namely, the time needed for the thin film to attain zero thickness at some point. William and Davis [2] examined the nonlinear evolution equation and numerically treated it as an initial value problem with periodic boundary conditions. Their results indicated that the nonlinearities of the system will accelerate the rupture phenomenon.

Kim and Bankoff [3] studied the interaction of an electrostatic field with a thin liquid film flowing under gravity down an inclined plane. They did not include the van der Waals force term in their analysis because they were concerned with thick films. They examined the stability and evolution of the interface in the thin film limit. A discussion was provided on the application of their numerical results to a proposed electrostatic liquid film space radiator.

A review of the literature indicated that no one until now has addressed the question of how the thin liquid film and an electrostatic field interact. The present work has been undertaken in order to investigate this problem. We are interested in the specific working regimes of the parameters, where it will be possible to stop rupture or dry out of the thin film. This will be accomplished by solving the equations of thin film motion in the presence of an electric field. A long wave theory is formulated for the nonlinear dynamic instabilities of the thin film.

ANALYSIS

We consider the flow of a thin liquid film down an inclined plane under gravity. The plane is assumed to make an angle β with the horizontal. We choose x and y directions to be parallel and normal to the plane, respectively as shown in Figure 1. We assume that the characteristic thickness of the film to be d and the length scale parallel to the film to be L. The aspect ratio $\xi = d/L$. For a thin film, $\xi << 1$. The distance from the charged foil and the plane is H.

The electric field is determined by solving Laplace's equation.

$$\nabla^2\phi = 0 \tag{1}$$

where $\phi(x,y)$ is the electric potential. The boundary conditions are

$$\phi(x,H) = \Phi(x); \quad \phi(x,0) = 0 \tag{2}$$

Along $y = h(x,t)$ we have the boundary conditions that the tangential electric field and the normal displacement field are continuous. It may be noted that $y = h(x,t)$ is unknown, so that solution of the electrostatic problem is coupled to the dynamics of the film.

The liquid film is governed by the Navier-Stokes equations. The liquid layer is assumed thin enough that van der Waals forces are effective and thick enough that a continuum theory of the liquid is applicable.

We assume that the liquid is incompressible. The governing equations and boundary conditions are made dimensionless by using the following scales: length in y-direction $\approx d$, in x-direction $\approx L$, velocity in x-direction $\approx U_0$, velocity in y-direction $\approx \xi U_0$, unit of time $\approx L/U_0$, unit of pressure $\approx \rho U_0^2$, and unit of electric field

$\approx F$. We take ρ as the fluid density, ε_0 the dielectric constant and μ the viscosity. The continuity equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

The momentum equation becomes

$$\xi\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\xi\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\xi^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{1}{Fr^2}\sin\beta-\xi\frac{\partial \psi}{\partial x} \tag{4}$$

$$\xi^2\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+\xi v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\xi}{Re}\left(\xi^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{1}{Fr^2}\cos\beta-\xi\frac{\partial \psi}{\partial y} \tag{5}$$

In the above equations, u and v are the velocity components in x and y directions respectively, p is the pressure and ψ is the dimensionless potential function representing the van der Waals forces. We follow Williams and Davis [3] and write a modified expression for ψ :

$$\psi = Ah^{-3} \tag{6}$$

where A is related to the Hamaker constant A' as $A=\frac{A'}{6\pi\rho U_0{}^2}$. We have introduced the Reynolds number, $Re=\rho U_0 d/\mu$ and the Froude number, $Fr=U_0/\sqrt{gd}$.

The boundary conditions along the solid plane wall are given by :

$$y=0\ ;\ u=v=0 \tag{7}$$

At the fluid interface, we have the kinematic condition:

$$y=h(x,t):\ \frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}=v \tag{8}$$

The continuity of tangential stress on the interface requires

$$y = h(x,t): \left[1-\xi^2\left(\frac{\partial h}{\partial x}\right)^2\right]\left(\frac{\partial u}{\partial y}+\xi^2\frac{\partial v}{\partial x}\right)+2\xi^2\frac{\partial h}{\partial x}\left(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}\right)=0 \qquad (9)$$

The continuity of normal stress at the interface y = h(x,t) becomes

$$\frac{\xi^2}{Ca}\frac{\partial^2 h}{\partial x^2}\left[1+\xi^2\left(\frac{\partial h}{\partial x}\right)^2\right]^{-3/2}+Ah^{-3}=-\frac{Re}{2}p+K\left(\frac{1}{\xi_f}-1\right)$$
$$\left[\left(E_n^v\right)^2+\xi_f\left(E_t^v\right)^2\right]+\xi\left[\xi^2\left(\frac{\partial h}{\partial x}\right)^2\frac{\partial u}{\partial x}-\frac{\partial h}{\partial x}\left(\frac{\partial u}{\partial y}+\xi^2\frac{\partial v}{\partial y}\right)\right]\times\left[1+\xi^2\left(\frac{\partial h}{\partial x}\right)^2\right]^{-1} \qquad (10)$$

where:

$$K=\frac{\varepsilon_0 dF^2}{16\pi\mu U_0}$$

ε_f = dielectric constant of the fluid (dielectric constant for vapor is taken as unity)

ε_0 = electrical permittivity of free space

$E_{n,t}^v$ = normal and tangential components of the electric field in the vapor at the interface

$Ca = 2\mu U_0 / \sigma$ is the capillary number

Equations (3)-(10) determine the motion of the liquid film. Our aim here is to solve for the stability of the liquid film while including the effect of van der Waals forces and an applied electric field.

We now apply the long-wave theory to study the stability problem. When the layer is thinner than a critical value, small disturbances begin to grow. These waves have wavelengths much larger than the mean thickness of the layer. Defining a small parameter κ that is related to wave number of such disturbances, we may rescale the governing equations:

$$X = \kappa x;\quad Y = y;\quad \tau = \kappa t \qquad (11)$$

We now assume the following expansions for the flow field:

$$\begin{aligned} u &= u_0 + \kappa u_1 + \kappa^2 u_2 + 0(\kappa^3) \\ v &= \kappa[v_0 + \kappa v_1 + \kappa^2 v_2 + 0(\kappa^3)] \\ p &= \frac{1}{\kappa}[p_0 + \kappa p_1 + \kappa^2 p_2 + 0(\kappa^3)] \\ \psi_0 &= \kappa\psi = 0(1) \text{ as } \kappa \to 0 \end{aligned} \tag{12}$$

Neglecting the variation of p with y and substituting expressions (12) into equations (1)-(10), we get

$$u_0 = \left[-\frac{Re}{Fr^2}\sin\beta + Re\cdot\xi\cdot\left(\frac{\partial p_0}{\partial X} + \frac{\partial \psi_0}{\partial X}\right)\right]\cdot\left[\frac{Y^2}{2} - hY\right] \tag{13}$$

$$v_0 = -Re\cdot\xi\cdot\left(\frac{\partial^2 p_0}{\partial X^2} + \frac{\partial^2 \psi_0}{\partial X^2}\right)\left[\frac{Y^3}{6} - \frac{hY^2}{2}\right] + \frac{Re}{2}\left[\frac{-\sin\beta}{Fr^2} + \xi\left(\frac{\partial p_0}{\partial X} + \frac{\partial \psi_0}{\partial X}\right)\right]\left(\frac{\partial h}{\partial X}\right)Y^2 \tag{14}$$

$$p_0 = \frac{2}{Re}\cdot\left(\overline{K}\left(\frac{1}{\varepsilon_f} - 1\right)[(E_m^v)^2 + \varepsilon_f (E_t^v)^2] - \frac{\xi^2}{\overline{Ca}}\cdot\frac{\partial^2 h}{\partial X^2}\right) \tag{15}$$

where $\overline{K} = K\cdot\kappa$; $\overline{Ca} = Ca/\kappa^3$.

Similarly, expressions u_1, v_1 and p_1 may be derived. Since these expressions are very long, they are not reproduced here.

Using equations (13)-(15) we may show that the leading order evolution equation for the film rupture is given by

$$\frac{\partial h}{\partial \tau}-\frac{h^2}{2}\left\{-\frac{Re}{Fr^2}\sin\beta+Re\cdot\xi\left[\frac{2}{Re}\cdot\left(-\frac{\xi}{\overline{Ca}}\cdot\frac{\partial^3 h}{\partial X^3}+\overline{K}\left(\frac{1}{\varepsilon_f}-1\right)\frac{d}{dX}[(E_n^v)^2+\varepsilon_f(E_t^v)^2]\right)\right.\right.$$
$$\left.\left.-\frac{3\kappa A}{h^4}\cdot\frac{\partial h}{\partial X}\right]\right\}\frac{\partial h}{\partial X}$$
$$=\frac{Re\xi h^3}{3}\cdot\left\{\frac{2}{Re}\cdot\left(-\frac{\xi}{\overline{Ca}}\cdot\frac{\partial^4 h}{\partial X^4}+\overline{K}\left(\frac{1}{\varepsilon_f}-1\right)\frac{d^2}{dX^2}[(E_n^v)^2+\varepsilon_f(E_t^v)^2]\right)\right.$$
$$\left.+\frac{3\kappa A}{h^4}\left[-\frac{\partial^2 h}{\partial X^2}+\frac{(N+1)}{h}\left(\frac{\partial h}{\partial X}\right)^2\right]\right\} \qquad (16)$$
$$+\frac{h^2}{2}\cdot\frac{\partial h}{\partial X}\cdot\left\{-\frac{Re}{Fr^2}\sin\beta+Re\cdot\xi\cdot\left[\frac{2}{Re}\cdot\left(-\frac{\xi}{\overline{Ca}}\cdot\frac{\partial^3 h}{\partial X^3}+\overline{K}\left(\frac{1}{\varepsilon_f}-1\right)\frac{d}{dX}[(E_n^v)^2+\varepsilon_f(E_t^v)^2]\right)\right.\right.$$
$$\left.\left.-\frac{3\kappa A}{h^4}\cdot\frac{\partial h}{\partial X}\right]\right\}$$

subject to initial conditions:

$$h(X,0)=F(X) \qquad (17)$$

Equations (16) and (17) may be solved numerically in order to predict the rupture characteristics.

We now define

$$Z=\left(\frac{ACa}{\xi}\right)x$$
$$T=\left(A^2Ca\right)t \qquad (18)$$

Equations (16) and (17) now reduce to the following form:

$$\frac{\partial h}{\partial T}+\frac{Re\sin\beta}{Fr^2}\cdot\left(\frac{1}{\xi A^3 Ca}\right)^{1/2}\cdot h^2\frac{\partial h}{\partial Z}-\frac{2K}{A}\left(\frac{1}{\varepsilon_f}-1\right)h^2\frac{\partial h}{\partial Z}\frac{\partial}{\partial Z}[(E_n^v)^2+\varepsilon_f(E_t^v)^2]$$
$$-Re\cdot h^{-2}\left(\frac{\partial h}{\partial Z}\right)^2+2h^2\frac{\partial h}{\partial Z}\cdot\frac{\partial^3 h}{\partial Z^3}$$
$$=-\frac{2}{3}h^3\frac{\partial^4 h}{\partial Z^4}+\frac{Re}{3}\cdot\frac{K}{A}\cdot\left(\frac{1}{\varepsilon_f}-1\right)h^3\cdot\frac{\partial^2}{\partial Z^2}[(E_n^v)^2+\varepsilon_f(E_t^v)^2] \qquad (19)$$
$$-Re\cdot h^{-1}\frac{\partial^2 h}{\partial Z^2}$$

with initial conditions:

$$h(Z,0)=g(Z) \qquad (20)$$

Equation (19) governs long wave interfacial disturbances to the static film (having h=1) subject to van der Waals attractions.

To linearize (19), we put h=1+H(Z,T) and the resulting linearized equation is given by:

$$H_T+\left\{\frac{Re\sin\beta}{Fr^2}\cdot\left(\frac{1}{\xi A^3 Ca}\right)^{1/2}-\frac{2K}{A}\left(\frac{1}{\varepsilon_f}-1\right)\cdot\frac{\partial}{\partial Z}\left[(E_n^v)^2+\varepsilon_f(E_t^v)^2\right]\right\}\cdot H_Z \qquad (21)$$
$$+ReH_{ZZ}+\frac{2}{3}H_{ZZZ}=0$$

RESULTS AND DISCUSSION

The nonlinear partial differential equation (19) was solved numerically using the finite difference method. Central differences were used for space variable and the midpoint rule was used for time. The Newton-Raphson method was used to solved the resulting system of difference equations. The problem was treated as an initial value problem with spatial periodic boundary conditions within the interval $0 < Z < q_m$, where $q_m = \frac{1}{\sqrt{2}}$ is the maximum wave number. We used spatial grid point N=50 and time steps $\Delta T = 0.001$ in all the computations. The initial condition was given by

$$h(Z,0) = 1 + 0.1\sin(q_m Z) \tag{22}$$

All the computations were done for a horizontal plate ($\beta = 0$).

Figure 2 shows the initial disturbance introduced and the film profile at the time of film rupture. Figure 3 displays the timewise variation of minimum film thickness with and without the imposed electric field effect. When electric field is imposed, the dielectric constant ε_f has been chosen as a parameter. It may be observed that the rupture time increases when electric field is imposed. The transient variation of minimum film thickness for ε_f=50(water), 81(glycerin) and ∞ coincide within the scale of the figure. As ε_f decreases to 2.5(trichloromonofluoromethane R11), the rupture time increases furthermore.

Figures 4-6 show the variation of the rupture time with the wave number of disturbance, q. From figure 4, we observe that the mode of the wave number corresponding to the minimum value of rupture time does not change significantly as the

value of ε_f changes. The wave numbers of these unstable modes are approximately equal to $1/\sqrt{2}$. When ε_f=2.5, the highest values of rupture times are obtained. From figure 5 and 6, we observe that for a given fluid, the ruptures times increase as the intensity of the electric field strength is increased.

CONCLUDING REMARKS

In this paper, we have formulated a long wave theory for the nonlinear dynamic instabilities of a thin liquid film interacting with an imposed electric field. Numerical solutions are obtained for the Navier Stokes equations governing the dynamics of the liquid film in the presence of an electric field. The results indicate that the film rupture time may be delayed by increasing the electric field strength.

ACKNOWLEDGMENTS

The authors are grateful to Mr. Chris Clay, Chief of Structural Integrity Branch at Wright Patterson Air Force Base for the financial arrangement to make this investigation possible.

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3. Kim, H. and Bankoff, S.G., "The Effect of an Electrostatic Field on Film Flow Down an Inclined Plane," Physics of Fluids, Vol. 4, 1992, pp. 2117-2130.

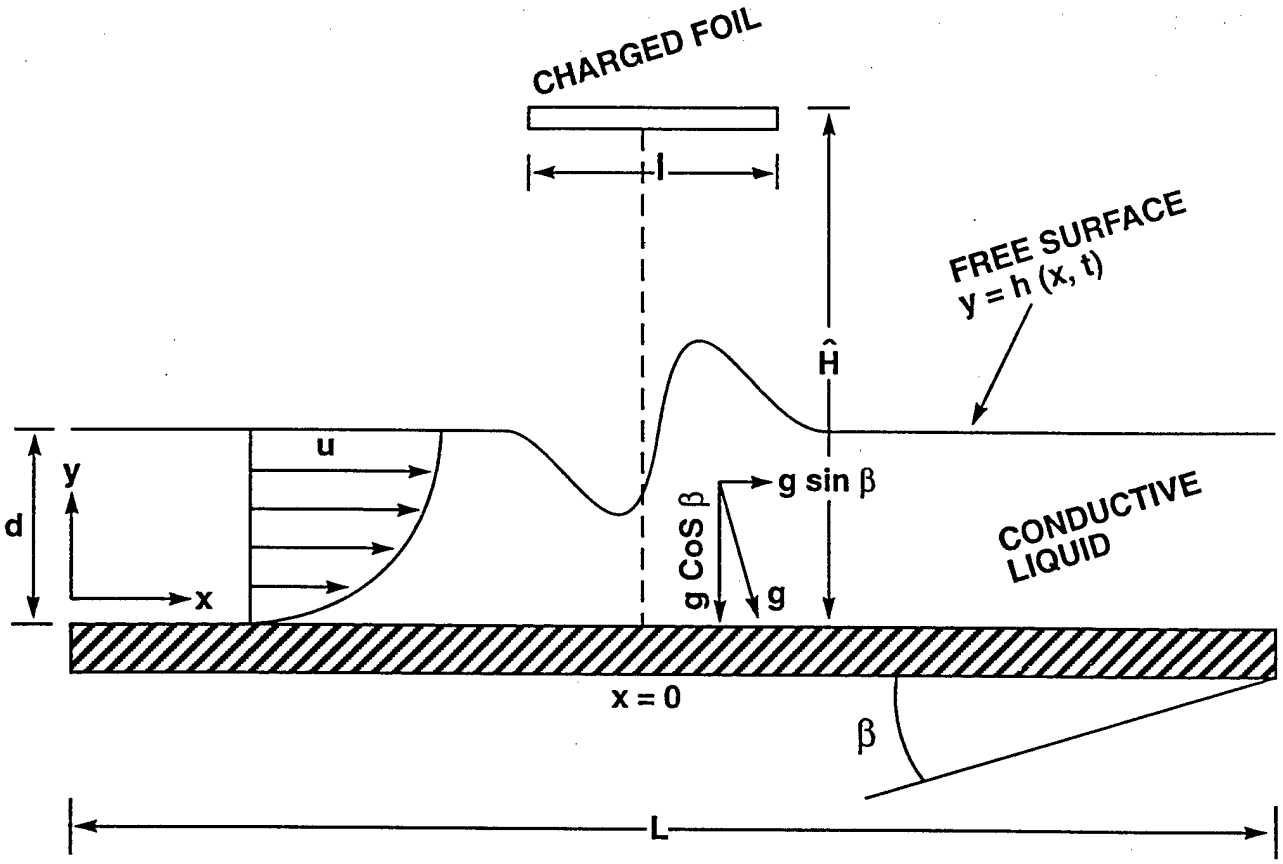

Figure 1. Flow Model for the Thin Film Flow

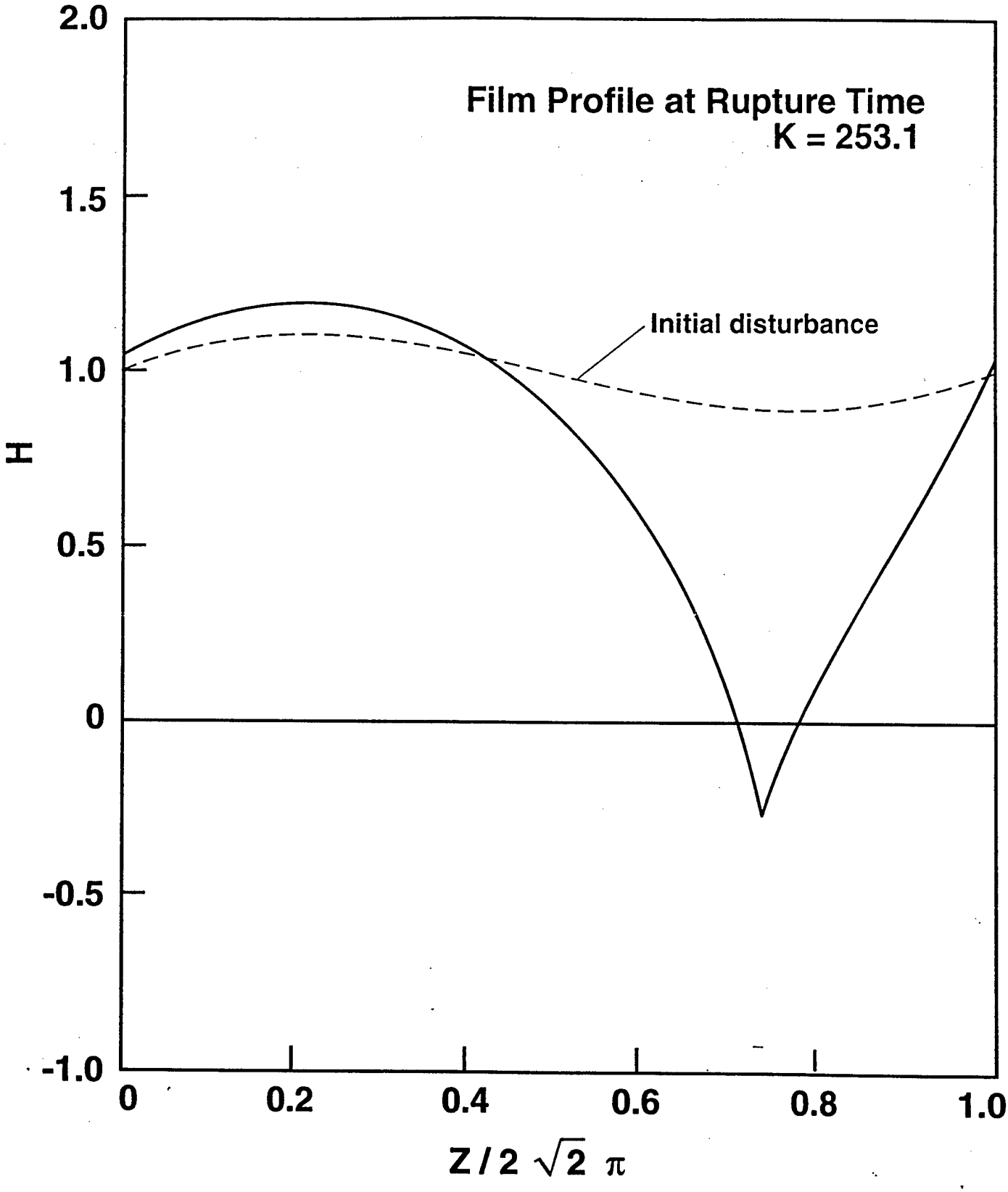

Figure 2. Film Thickness Distribution at the Rupture Time

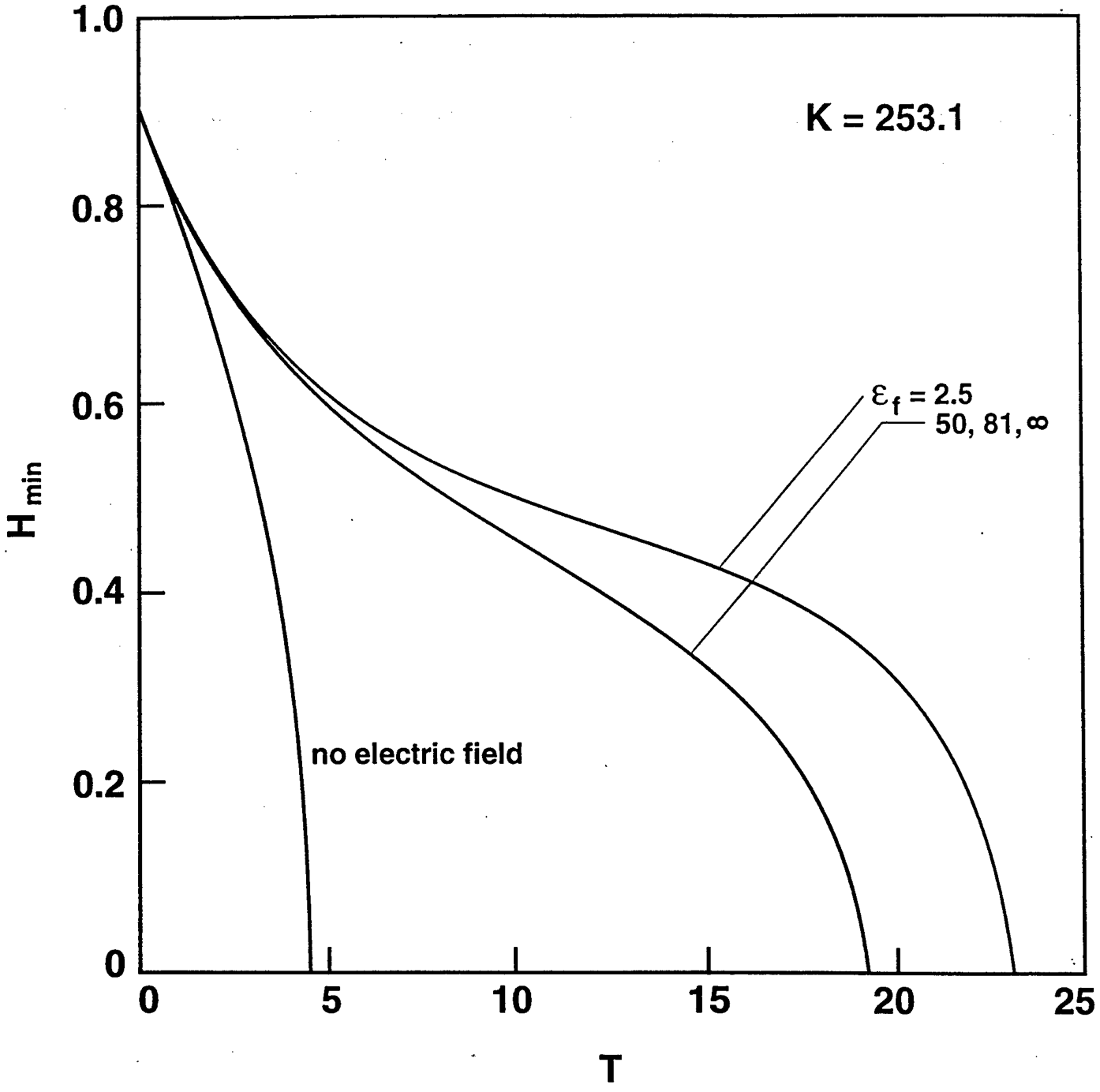

Figure 3. Minimum Film Thickness Versus Time

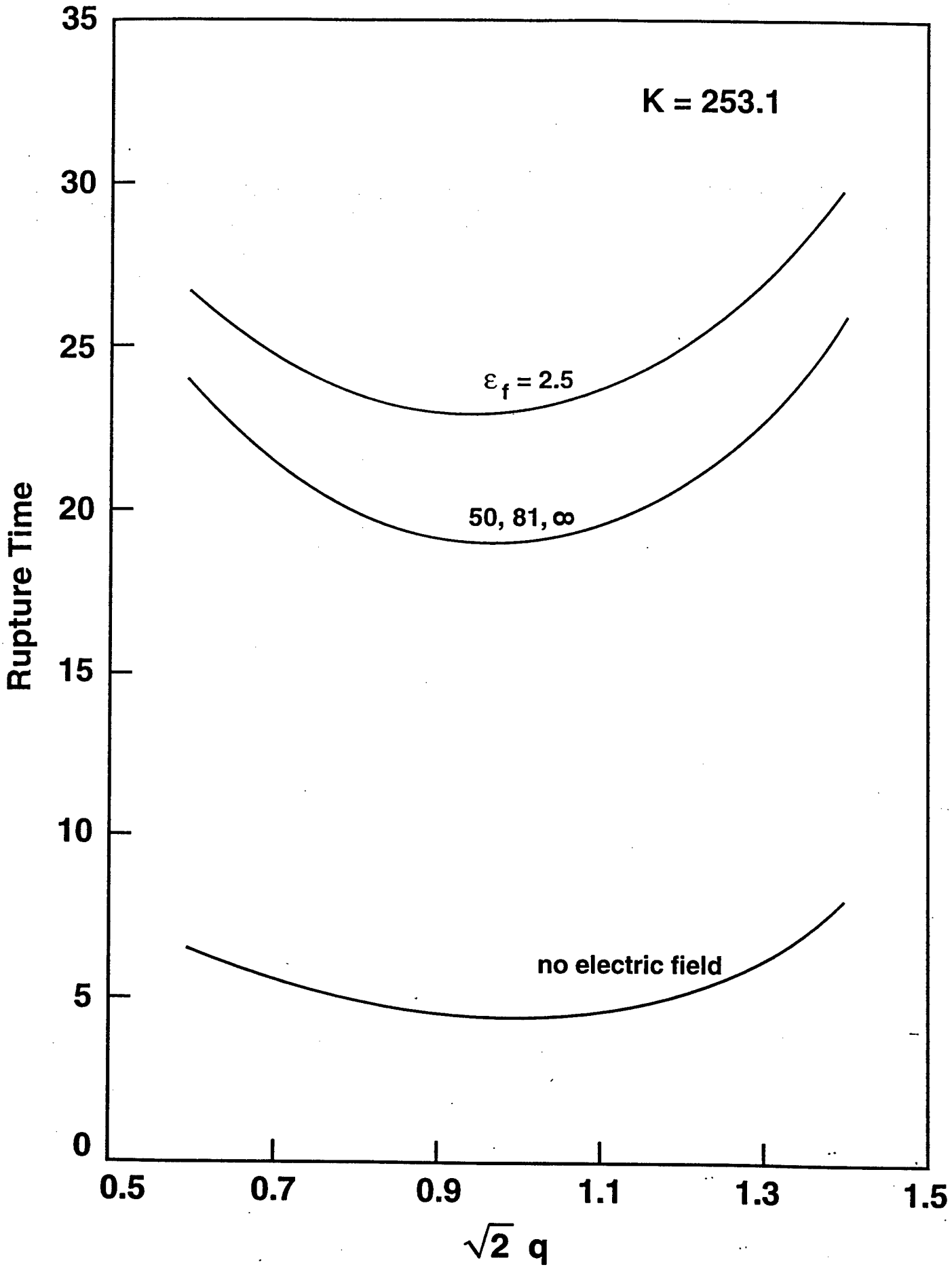

Figure 4. Rupture Time Versus q(k = 253.1)

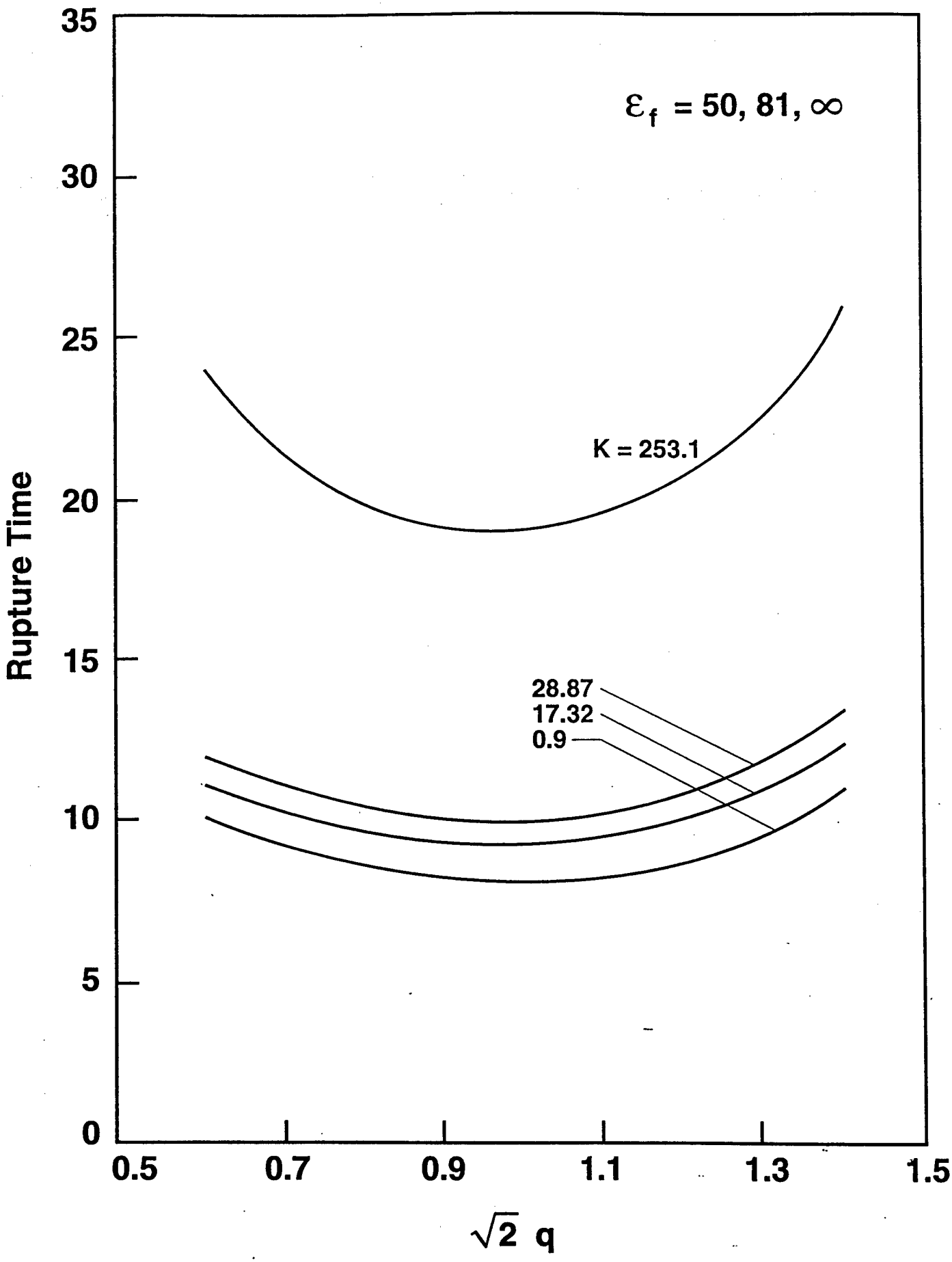

Figure 5. Rupture Time Versus q(ε_f = 50,81,∞)

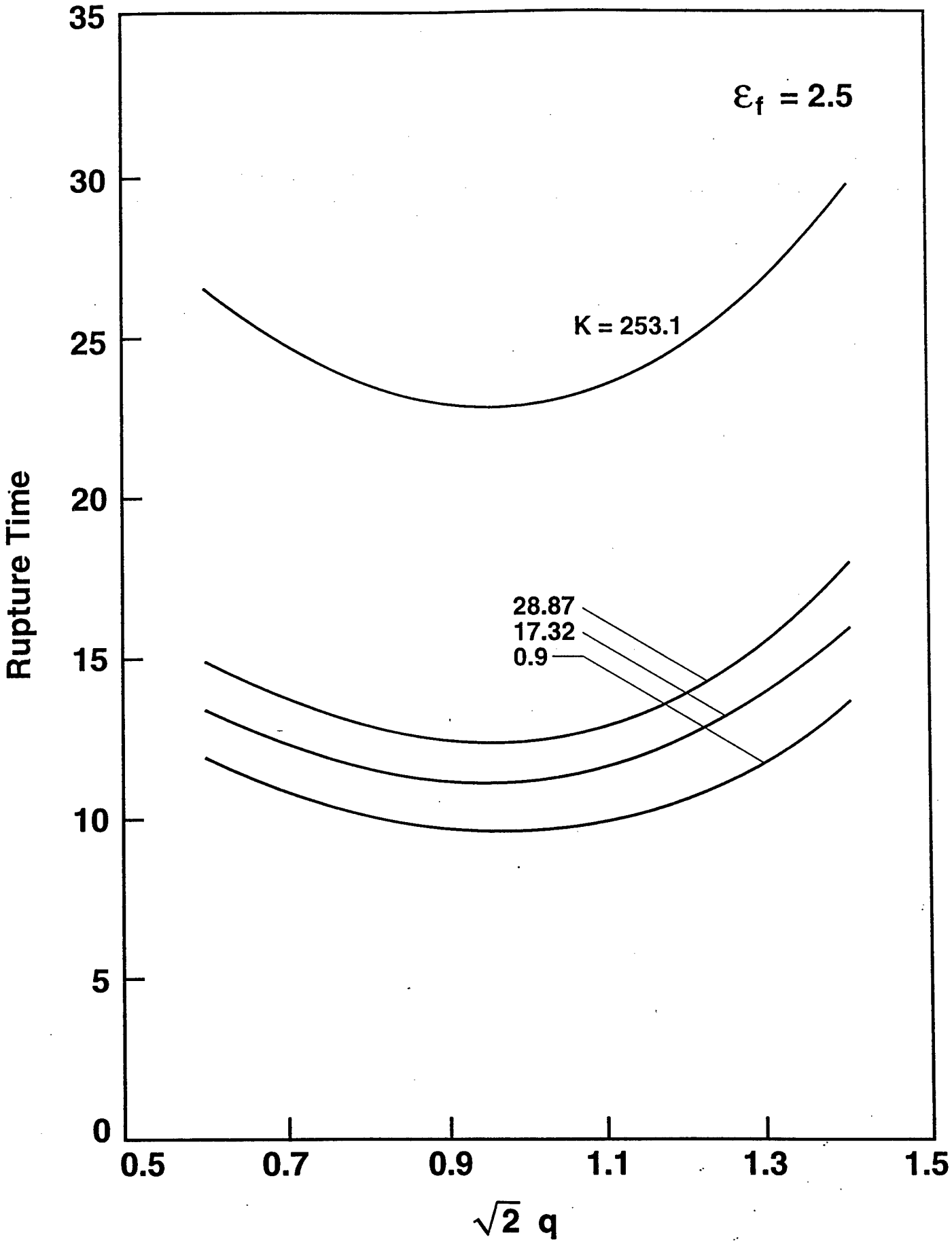

Figure 6. Rupture Time Versus q(ε_f = 2.5)